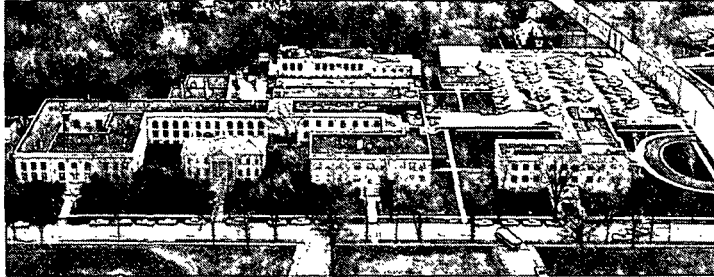


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THE ULTIMATE STRENGTH OF PAPER

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THE ULTIMATE STRENGTH OF PAPER

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THE ULTIMATE STRENGTH OF PAPER

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1.0 Introduction

Understanding and predicting the ultimate strength of a material is one of the most challenging problems facing the materials scientist. Considerable progress has been made toward predicting the elastic, viscoelastic and yield behavior of materials, while failure properties are much more intractable as well as being largely empirical. Paper is a complex material and understanding its behavior at a fundamental level is a formidable task; nevertheless, there is a substantial body of knowledge which contributes to our understanding of the ultimate strength of paper in terms of its dependence on raw materials and papermaking process variables.

One material is distinguished from another by the type and arrangement of its constituent atoms and molecules. Where this arrangement is regular or ordered the material is said to be crystalline or polycrystalline. When there is a lack of order the material is said to be amorphous. Cellulose, a naturally occurring polymer, is comprised of crystalline and amorphous regions, or as some would prefer, ordered and less ordered regions. Obviously the ultimate strength of a material will depend on organization at the atomic or molecular level. See for example, Kelly (1). However, other organizational levels may be more critical e.g., crystal size in polycrystalline materials, fiber and matrix properties in composite materials, (2) and pore size and distribution in porous materials (3).

In broad terms the deformation and failure of a material may be characterized as being either brittle or ductile. One approach to characterizing the deformation and failure of polycrystalline materials is given by Ashby (4). The mechanism of failure and the region over which it is applicable is shown on a fracture map. A fracture map for ice, a hydrogen bonded polycrystalline solid, developed by Ashby is shown in Figure 1.

According to Ashby (4), following Kelly (1) and others, the ideal or maximum strength of many materials is equal to one tenth of its elastic modulus E , i.e. $\sigma/E \approx 0.1$. This limit then forms the upper boundary of the map, and the ordinate is also designated σ/E . The melting temperature, T_m , is closely related to the cohesive strength of the material. The temperature, T , at which the strength determination is made will also influence the type of fracture produced, i.e. brittle or ductile. Therefore, as seen in Fig. 1, the abscissa of the map is T/T_m . At low temperatures where brittle fracture occurs, i.e., for ice $T/T_m < 0.6$ there are two types of fracture shown: cleavage 1 and cleavage 2. Cleavage 1 is initiated by natural or induced flaws in the material, and the upper limit of this region is set by Griffith's equation for brittle fracture:

$$\sigma = \left(\frac{E G_c}{\pi C} \right)^{\frac{1}{2}} \quad (1)$$

where E , G_c and $2c$ are respectively the elastic modulus, fracture toughness, and crack length. Cleavage 2 type fracture occurs for a relatively crack free material but now the mechanism is one of slip or twinning within the grain which can nucleate cracks. The fracture is still brittle but is preceded by a zone of microplasticity. A recent review of fracture physics is given by Lawn (5).

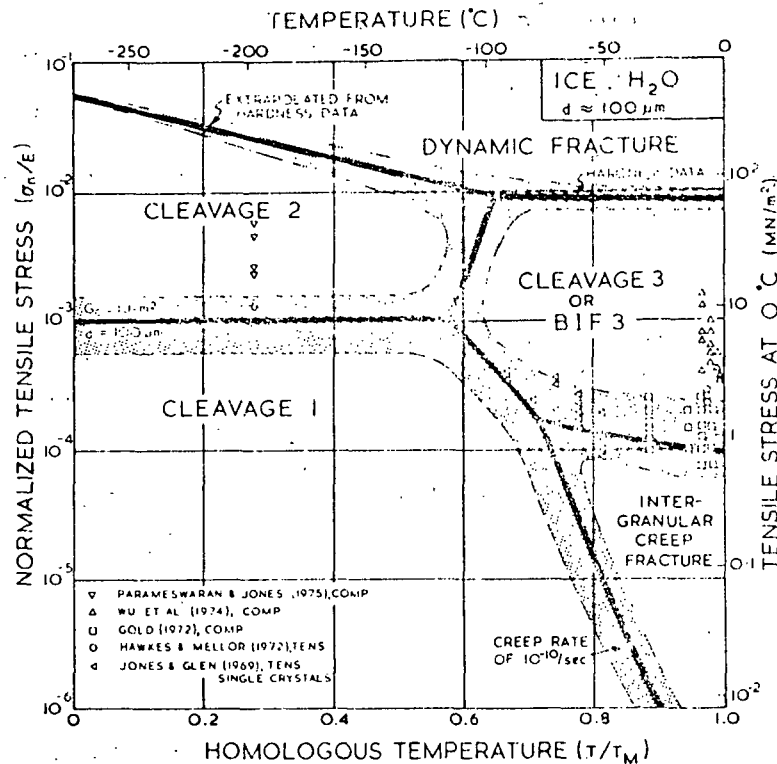


Figure 1. Fracture map. [Fig. 31, p. 166. Ashby (4)]

Ashby (4) has drawn fracture maps for a wide range of polycrystalline materials including metals, ceramics, and ice. It should in principle be possible to construct some form of fracture map for polymeric materials. Of course they are only as accurate as the results from which they are drawn and observations of the fracture surface. One important, but perhaps obvious point made by these maps is that the type of fracture is dependent on the regime in which one is operating.

Cellulose fiber fracture is quite complex. See, for example, the photomicrographs of Harada and Cote, p. 441 (6). As far as I am aware of, no extensive studies in this area have been published. Furakawa's Ph.D. work (7) on the fracture of single fibers will be published shortly.

Brittle fracture, i.e., cleavage 1 is the only one at the moment which is amenable to analysis, i.e., Griffith theory, and is characterized by low values of fracture toughness as shown by the ranges calculated by Ashby given in Table I.

TABLE I
CALCULATED VALUES OF FRACTURE TOUGHNESS. Ashby (4).

Material	G_c (2 x surface energy) J/m ²
metals	0.5-3
ceramics	0.5-4
polymers	0.1-1

As the material becomes more ductile, i.e., with increasing temperature, fracture toughness increases dramatically and may be in the range of 10^3 to 10^7 J/m². (As we shall see later values of fracture toughness for paper are around 10^4 J/m² at standard testing conditions.)

So far we have been discussing continuum single phase polycrystalline materials, and when conditions for brittle fracture are satisfied the relationship between maximum tensile stress and crack length is in good agreement with Equation 1. Another interesting area of materials science is the development of high strength cements (8) known as macro defect free cements (MDF). The very significant strength improvements obtained are attributed to a reduction in flaw size. This explanation has recently been challenged by Eden and Bailey (9), and although they find that the elimination of voids and defects does yield a modest strength improvement, it is suggested that the addition of polymer, which is part of the MDF cement process, may be responsible for the large strength improvement found.

It is not always possible or desirable to have completely non-porous materials. Therefore, at another level of organization we are interested in maximizing the strength of a variety of materials at different porosity levels. These materials include paper, sintered metals, porous plastics, foams, etc. It can be anticipated that the strength of a porous material will be less than one without voids even when some sort of allowance is made for reduction in load bearing area (this is not always explicitly stated in published results.)

Some of the factors which might be important in controlling the strength properties of porous materials include:

- 1) The size, shape and distribution of pores.
- 2) The structure of the material and how it might differ from a non-porous material of the same composition.

- 3) The actual minimum load bearing area.
- 4) The bonding which exists between the structural elements comprising the porous medium.
- 5) The size, shape, and distribution of the structural elements comprising the porous medium.

As one might expect from the number of complications which arise when considering the strength properties of porous materials, no good theoretical foundation for their prediction has yet been developed. A number of empirical strength relationships, however, have been developed for concrete, (9) sintered metals (3,10) and paper (11,12,13). The variables include porosity, pore shape factor, and bonding. Birchall et al., (9) have developed an empirical equation for cement using Griffith's basic equation as follows:

$$\sigma = \left[\frac{E_0 G_{0C} (1-\epsilon)^3 \exp(-d\epsilon)}{\pi C} \right]^{\frac{1}{2}} \quad (2)$$

where E_0 and G_{0C} are the elastic modulus and fracture toughness at zero porosity, i.e., $\epsilon = 0$. d is an experimentally determined constant. We can interpret the equation as saying that the fracture toughness is effectively reduced by the porosity terms $(1-\epsilon)^3 \exp(-d\epsilon)$. For a given porosity ϵ the constant d must therefore account for some average stress concentration factor which might be expected to depend on pore size and shape. There is clearly a need to better understand the factors controlling the ultimate strength of porous materials.

To return to cellulose and paper, we are all familiar with the different organizational levels which exist from the basic cellobiose unit to the network arrangement of fibers which we recognize as paper as shown in Fig. 2.

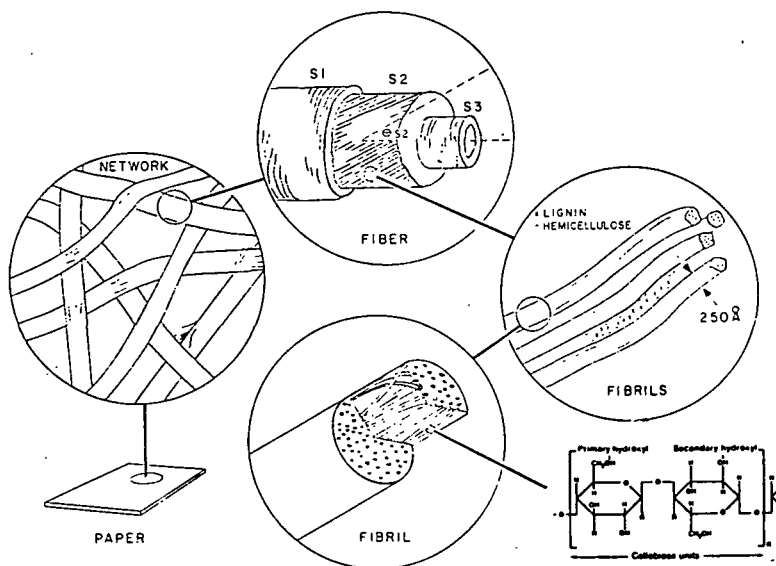


Figure 2. Paper at different organizational levels. Baum (13a).

We will try not to be intimidated by the complexity of this organization. Our hope is to gain sufficient understanding of it to facilitate our design approach. Table II summarizes elastic modulus, E , and tensile strength, σ , data for cellulose at various levels of organization. Both predicted and measured values are incorporated where appropriate.

TABLE II
MODULUS AND STRENGTH VALUES FOR CELLULOSE.

Level of Organization	Theory			Experiment		
	E , GPa	σ , GPa	σ/E	E , GPa	σ , GPa	σ/E
Crystalline Cellulose	134 ¹ 250 ⁴ 246 ⁴	19 300 ⁵ 19	0.007			
Crystalline cellulose embedded in matrix	175 ⁴					
Single Fiber						
wood fiber $\alpha=0$	175	13.5	0.077	76.9 ²	1.13 ³	0.0147
hemp ⁶ $\alpha=2.3$				70	0.92	0.013
jute ⁶ $\alpha \approx 7.9$				60	0.86	0.014
flax ⁶ $\alpha \approx 6.0$				95	0.84	0.0088
flax ⁶ wet				27	0.88	0.0326
ramie ⁶ $\alpha \approx 6.0$				80	0.92	0.0115
ramie wet				19	1.08	0.0568
Ideal Network $(1/3)E$ or $(1/3)\sigma$ wood fiber.				25.6	0.377	
Ideal Network $(1/3)E^*$ or $(1/3)\sigma^*$ " " Nm/g				8530	243	

α : fibril angle. 1. (14) 3. (16) 5. (6)
2. (15) 4. (17) 6. (18)

As one might expect, the ultimate strength values for the Bast Fibers are close to the value for the wood fiber. However, they are all considerably lower than the calculated or ideal values. It is interesting to note that σ/E predicted is approaching 0.1 while the measured values are around 0.01. This is probably due to the fact that these measurements were made under ductile conditions. Both modulus, strength, specific modulus, and specific strength have been calculated for an ideal network of wood fibers as shown in Table II. These values represent ideal or maximum values for networks of unmodified fibers with minimal defects. We will be making further comparisons of the data shown in Table II.

In this introduction the factors affecting the ultimate strength of materials and their dependence on organizational level have been briefly reviewed. In what follows we will define those paper properties which are of particular interest to us and review the methods for their measurement. This will be followed by an examination of how they are affected by environment, raw material, and paper-making process variables. Predictions for the ultimate strength properties of paper is an important aspect of the design approach, which we will also survey. Finally, we will examine the ultimate strength requirements of some paper products.

2.0 Strength Properties:

Strength tests for paper have been developed for two main reasons. The first is motivated by a need to develop a test which is directly related to the converting and/or end-use properties of a particular paper product. The second is to gain a more fundamental understanding of the strength behavior of paper in terms of the raw material and process steps used to produce it. In the first case the interpretation of the results is often times very difficult because of the complex stress situation to which the material is subjected.

In some instances the complex stresses which a material is subjected to during converting and end use can be resolved into combinations of simple stresses, although the effects may not be simply additive, particularly where large deformations or failure are involved. For example, in the production of corrugating medium, the medium is generally subjected to tensile, shear, and compressive stresses. Either alone or in combination these stresses may reach a level which will result in flute fracture. Even if the material does not fail, its end use performance may be impaired. In the case of medium we refer to these as forming losses which can result in a loss of flat crush or edgewise compressive strength as shown by Whitsitt (19). From a design point of view these losses have to be minimized, although it is recognized that they may not be totally eliminated.

The basic strength properties we shall consider are tensile, compressive, and shear and are defined in Table III.

TABLE III
SPECIFIC STRENGTH PROPERTIES OF PAPER AND BOARD

Strength Property	In-Plane	Out-of-Plane
Specific Tensile Strength or Tensile Index	σ^*	σ_z^*
Specific Compressive Strength or Compressive Strength Index	σ_c^*	-
Specific Shear Strength or Shear Strength Index	τ_{xy}^*	τ_{xz}^*, τ_{yz}^*

x: machine direction, y: cross machine direction, z: thickness direction
 σ : maximum tensile stress, τ : maximum shear stress, ρ : density, $\sigma^* = \sigma/\rho$.
 $\tau^* = \tau/\rho$.

The strength properties are actually expressed as the strength to weight ratio or specific strength. For example, the in-plane specific tensile strength σ^* is calculated as follows:

$$\sigma^* = \frac{\text{Ultimate load/unit sample width}}{\text{Basis Weight}} \quad (3)$$

in SI units it will be Nm/g.

This definition is straightforward, although a choice has to be made sometimes as to what constitutes load bearing material, i.e., it might be appropriate to exclude non-load bearing elements such as filler etc., depending on the comparisons one wishes to make. The definition of out-of-plane specific strength properties are by comparison somewhat more ambiguous.

3.0 Measurement of Strength Properties

The recent handbook edited by Mark (6) gives an excellent survey of techniques for measurement of the strength properties shown in Table III and emphasizes the growing importance of out-of-plane properties and combined stress situations.

In-plane Strength Properties:

Uniaxial Strength σ^*

In-plane measurements are easier to make and are less time consuming than out-of-plane measurements. Even uniaxial tensile measurements which are relatively simple to make require care and attention in sample preparation and cutting procedures (20). Normally when performing tensile tests it is usual to record load elongation behavior so that other properties such as initial modulus, elongation at maximum strength, and tensile energy absorption may be derived as shown in Fig. 3.

Care has to be taken to avoid sample slippage in the jaws if accurate strain measurements are to be made, and this can be achieved through the use of line clamps (21). Sometimes it is preferable to make external strain measurements rather than relying on jaw separation. See Chapter 4 (6). Techniques for more detailed and localized strain analysis include Interference Holography (22) and Moire methods (23).

In paper testing it has been expedient to use rectangular tensile specimens, although necked specimens are preferred in the testing of other materials, e.g., metals, plastics, etc. A recent analysis by Kimura and Shimizu (24) has shown that negligible error is incurred in making modulus measurements using rectangular samples, although stress nonuniformity in the clamping area is evident from their work and may on occasion be responsible for failures in that region.

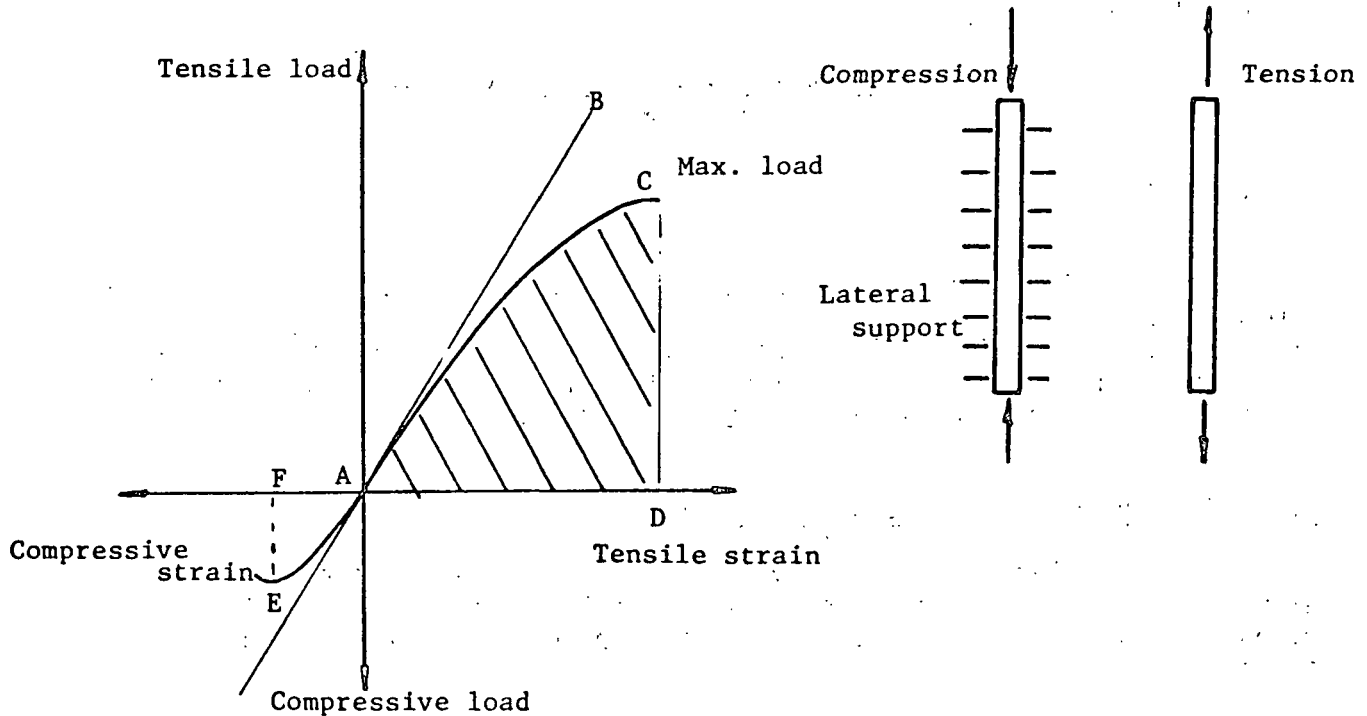


Figure 3. Load elongation curve for paper in tension and compression.

Edgewise compressive strength σ^*_c

There has been a good deal of interest and activity in recent years to measure the edgewise (in-plane) compressive strength of paper and board (25) - (30). Compressive strength measurements require some means of lateral support in order to suppress buckling as shown in Fig. 3. It has been found for a certain range of slenderness ratios that a plateau region exists where compressive strength is constant as shown in Fig. 4 and is referred to as the intrinsic compressive strength.

A number of methods have been developed to provide lateral support of the sample during compression testing and are shown in Figure 5.

It has been demonstrated that the form of support can have an important influence on the stress-strain curve in compression as well as ultimate strength. The method developed by S.T.F.I. avoids the problems associated with lateral support, and they have developed a very simple and straightforward method of measuring intrinsic compressive strength. A comparison of lateral support, and lateral support and short span methods of compressive strength measurement have been made by Seth and Soszynski, (27) and Seth (28), respectively. Gunderson (30) has also reviewed the three methods which have been developed at F.P.L.

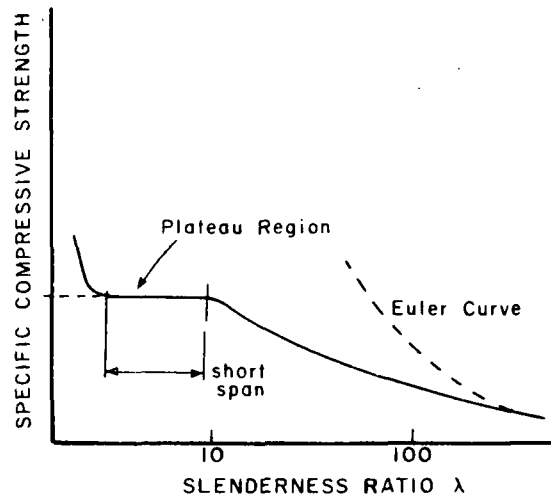


Figure 4. Variation of compressive strength with slenderness ratio.

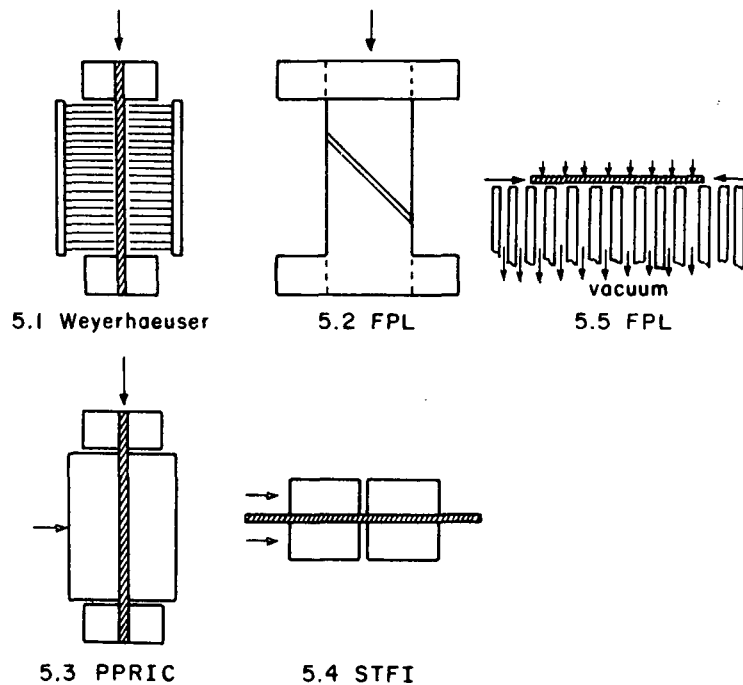


Figure 5. Methods of Lateral support for compression measurements.

It appears that measurement of in-plane compressive deformation and strength is in a fairly advanced stage of development and further effort will probably be concentrated on straightforward methods to accurately measure the stress-strain curve in compression.

In plane Shear τ_{xy}^*

In-plane shear is an important deformation mode, and the elastic properties can be readily measured using either ultrasonic (31) or mechanical (32,33) methods some of which are shown in Fig. 6.

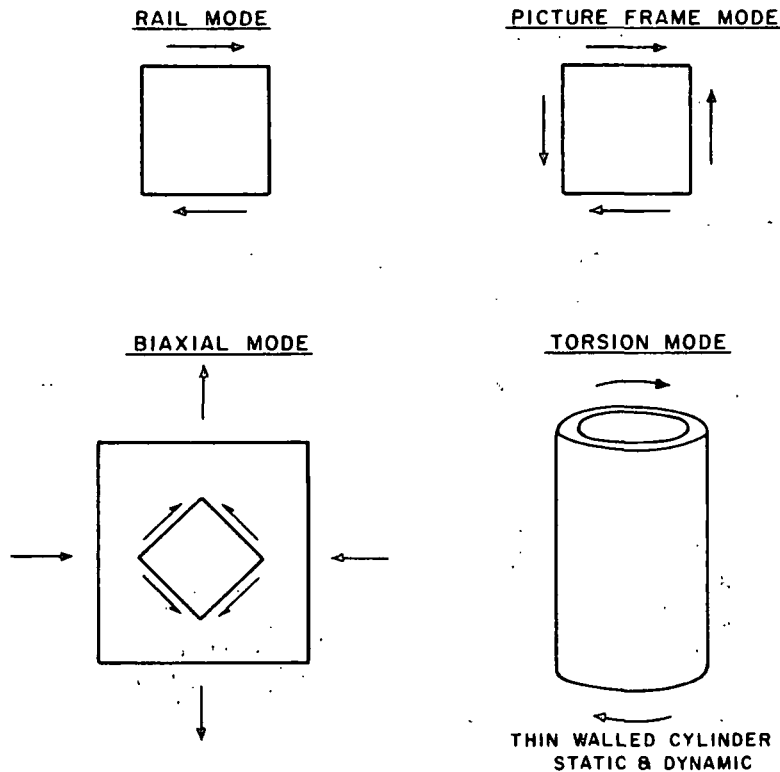


Figure 6. Mechanical methods of measuring in-plane shear deformation.

In mechanical testing large shear deformation is ultimately limited by an instability phenomenon known as shear buckling. As far as I am aware there have been no publications on the measurement of intrinsic in-plane shear strength of paper and board. As Bennett, et al. (6) point out, failure is likely to be in compression, since pure shear can be represented as a combination of tension and compression. Biaxial tension and compression have been employed to obtain a state of pure shear for making in-plane measurements (34,35). The value of an intrinsic shear strength measurement may be questioned, but, in principle, it should be possible. The device developed by Arcan, et al. (36) for generating a pure shear deformation mode shown in Fig. 7 is one possible approach.

Out-of-Plane Strength Properties

Measurement of out-of-plane deformation behavior is time consuming and more demanding than in-plane measurements, although elastic properties can now

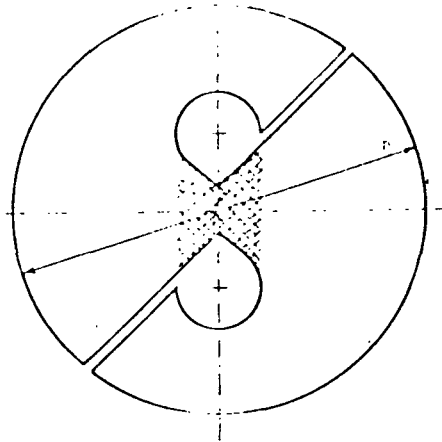


Figure 7. Device for Pure Shear Deformation. Arcan (36)

be readily measured using ultrasonic techniques (36a). Difficulties associated with this deformation mode include relatively small strains and finding suitable means for stress transmission to the sample such that its behavior is not modified or at least can be corrected for. Another problem area is the effect of property variations in the thickness direction which may result from inhomogeneities in composition, consolidation, and drying, to mention three important areas.

Z-Tensile σ_z^*

A few studies have been made of out-of-plane tensile or Z-direction tensile deformation behavior (37,38). Measurements of ultimate strength were made by Wink and Van Eperen. Considerable attention was paid to the factors and variables affecting its measurement, including sample surface preparation, adhesive amount, mounting and alignment.

Out-of-Plane Shear τ_{xz}^* , τ_{yz}^*

Out-of-plane shear deformation behavior was recently reviewed by this author (39), and a new technique for its measurement presented. Out of plane ultimate shear strength measurements have also been made by a number of researchers (40) - (43). Some configurations for out-of-plane strength measurements are shown schematically in Fig. 8. However, there still exists a need for a rapid and accurate measurement of out-of-plane shear deformation and strength.

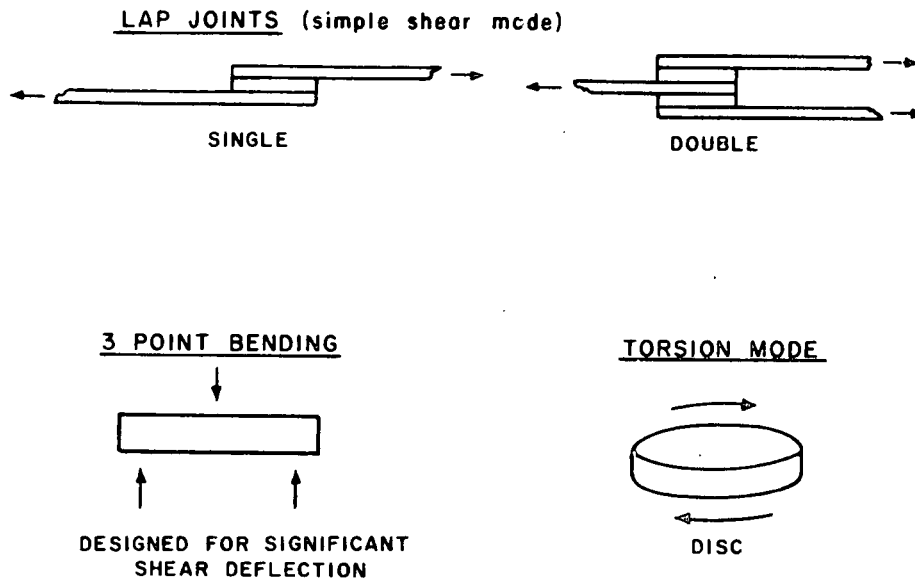


Figure 8. Configurations for shear strength measurement.

In a limited study a comparison of shear strength measurements made by Byrd of F.P.L., using his simple shear method, (40) and measurements made at I.P.C., using the torsion device (39) and the simple shear device shown in Fig. 9, are given in Table IV.

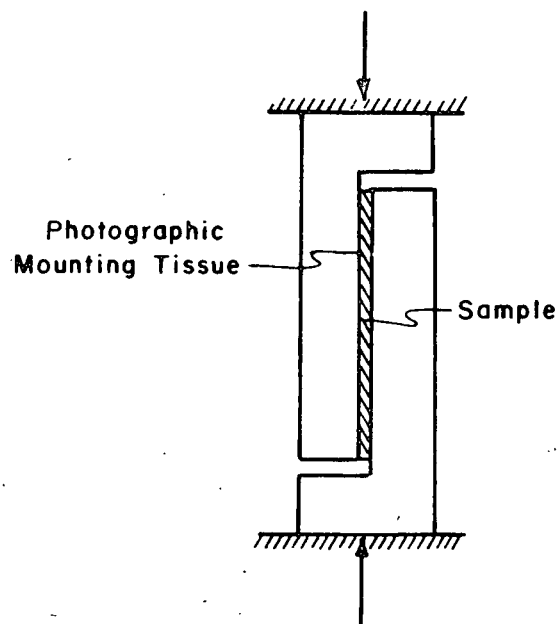


Figure 9. IPC configuration for shear strength measurement.

TABLE IV
COMPARISON OF FPL AND IPC SHEAR STRENGTH MEASUREMENTS

	FPL Simple Shear	IPC Simple Shear	IPC Torsion
Average max shear stress MD	2.03 MPa	2.00 MPa	-
Average max shear stress CD	1.85 MPa	1.61 MPa	-
MD/CD Ratio	1.10	1.24	
Mean Shear Strength			
$\bar{\tau} = \frac{2\tau_{xz}\tau_{yz}}{\tau_{xz} + \tau_{yz}}$	1.94 MPa	1.79 MPa	2.42 MPa

We see that there is reasonable agreement between the two simple shear measurement techniques, while the torsional method gives a higher value. This may be due to differences in stress concentration between the two methods or because the mode of failure is fundamentally different.

Effect of Environment on Strength Properties.

Moisture, temperature, frequency, or time scale of testing are perhaps the most important environmental variables affecting the strength properties of paper. In some applications aging and exposure to radiation may also be important. Paper will generally exhibit both elastic and plastic behavior, the relative amounts depending on composition and environmental conditions. Furthermore, we would expect following our earlier discussion of Ashby's work (4) that the failure of paper could also be categorized as either being ductile or brittle.

In cellulose, viscoelastic and plastic flow behavior will be influenced by the relative amounts of crystalline (ordered) and amorphous (less ordered) material present. A truly amorphous polymer has a well defined glass transition temperature T_g , and at temperatures below T_g the polymer exhibits glass-like behavior, while above it, rubbery behavior.

Glass transition or softening temperature ranges of some cell wall components given by Salmen (45) are shown in Table V.

TABLE V
SOFTENING TEMPERATURES. Salmen (45)

Cell wall Material	T_g
Cellulose	230°C
Hemicellulose	150-220°C
Lignin*	124-193°C

*Native lignin may be higher.

In the dry state at temperatures below T_g these components will behave as a glass. Moisture may be viewed as a plasticizer which can effectively lower the T_g or softening temperature of these components (45). At a given relative humidity the moisture content of paper will depend on the amount of amorphous cellulose present. It would therefore be more meaningful to compare properties at a given moisture content and temperature rather than at a fixed relative humidity and temperature. Again Salmen has shown as illustrated in Fig. 10 that the variation of elastic modulus with moisture content is independent of pulp type when based on amorphous content.

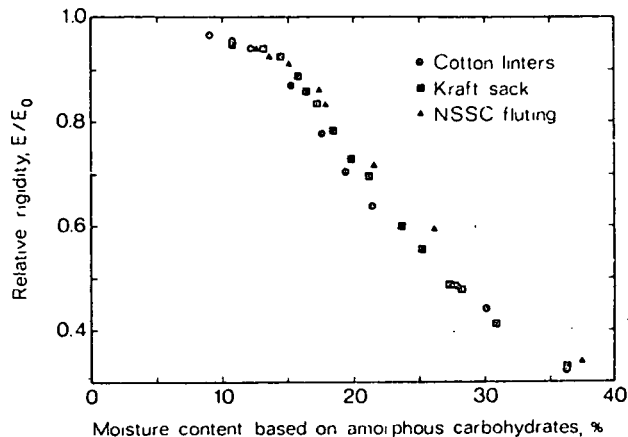


Figure 10. Variation of modulus with moisture content. Salmen (45)

An increase in moisture content generally results in a reduction in ultimate strength properties (45). This is usually attributed to a reduction in interfiber bonding rather than fiber tensile strength. Swelling effects may also increase stress concentration within the network. There has not been much published on the effects of moisture on wood fiber tensile strength. Cotton and the Bast fibers shown in Table II have higher wet tensile strengths. Our own experience (45a) with zero span strength, wet and dry, is inconclusive, and yet Cowan (46) indicates that extrapolated values of zero span strength, wet and dry, should be the same. As might be expected, increasing temperature has an adverse effect on strength properties as shown by Wink (47) and Salmen (48). Salmen suggests, following Zhurkov (49), that fracture is a thermally activated process and that tensile index should be inversely proportional to temperature.

4.0 The Influence of Raw Material and Papermaking Process Variables.

Raw material and papermaking process variables are obviously important factors determining the ultimate strength properties of paper. Their precise role will depend on the particular strength property under consideration and the failure mechanisms associated with it. Uniaxial tensile strength is the most commonly used indicator of paper strength. It is generally accepted that certain network, fiber, and interfiber bonding properties are the main variables controlling strength. Oversimplifying, we might say that interfiber bonding is responsible for realizing the deformation potential of the fiber.

If we can accept, for the moment, apparent density as a measure of inter-fiber bonding, then we find that all of the strength properties shown in table III increase with interfiber bonding. Many factors, i.e., raw materials, refining, wet pressing, and drying, influence network densification. The flexural rigidity (EI) of the wet fiber is perhaps the most important variable controlling densification, and the influence of species, morphology, pulping process and geometry on it are reasonably well understood. The flexibility of individual fibers may be modified by refining, a process we identify as internal fibrillation. This change has been demonstrated by a number of researchers (50) - (53). After formation the network is wet pressed and consolidated by external mechanical forces and internal forces generated by capillary action.

The manner in which the sheet is consolidated and the means used to aid water removal, i.e., the felt type, can also effect strength properties. This is illustrated in Fig. 11 where we see that a commercial wet press felt gives lower handsheet strength properties than those obtained with standard blotter stock (Waterhouse 53(a)).

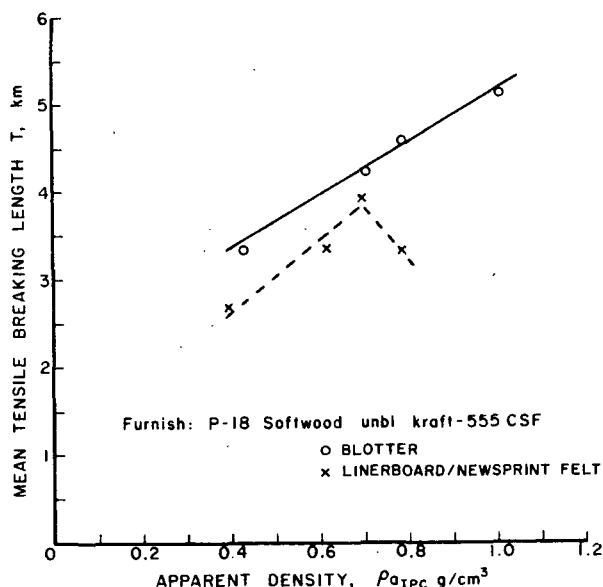


Figure 11. The effect of wet press felt type on strength.

The manner in which paper is restrained during drying can have a profound effect on its properties. Lateral shrinkage of individual fibers is transmitted through interfiber bonding and will result in significant sheet shrinkage from the wet to the dry state, unless restrained from doing so.

Restraint during drying can affect fiber properties, particularly at the interfiber bond regions, and final density. Baum, et al. (54) have shown that wet straining followed by complete restraint during drying reduces sheet density. We have also found (44) that the density of freely dried sheets is higher than sheets dried under full restraint. Therefore, we expect sheet density to vary with drying restraint conditions as shown in Fig. 12.

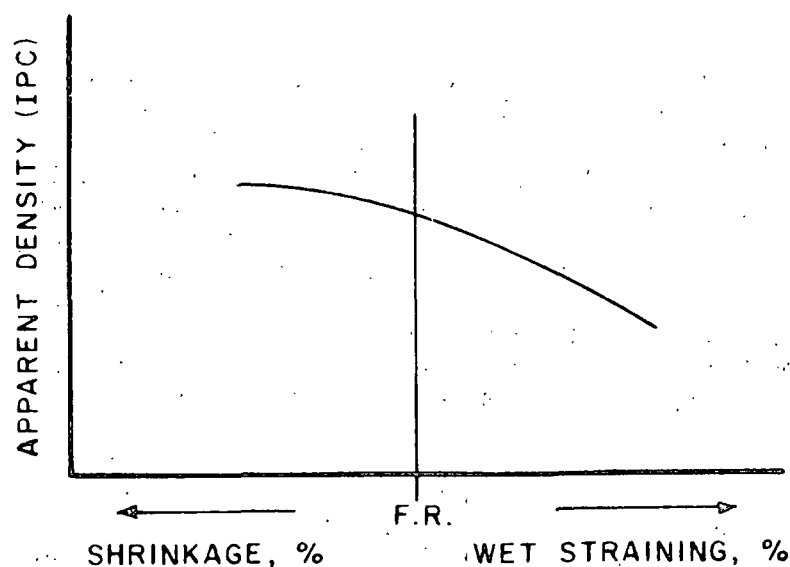


Figure 12. Variation of sheet density with drying restraint.

Our understanding of the deformation behavior of single wood pulp fibers, including the effects of drying, has been greatly increased in recent years. These findings are important to our appreciation of the role of fiber deformation in a network even though they may have undergone considerable modification by the paper making process. Fiber defects, i.e. microcompressions and curl induced during pulping, bleaching, or refining can also affect ultimate properties. These defects, if sufficient bonding is present, can result in increased elongation at rupture, although tensile strength will usually be lower. This can be of considerable importance in improving wet web performance (55) and the deformation of polymer impregnated networks (56).

Restraint during drying can effectively modify fiber deformation behavior. The deformation behavior of interfiber bonds and segments has been studied by Giertz (57). From another point of view Htun (58) has proposed that strength and related properties are directly dependent on the level of internal stress in the network as shown Fig. 13. Htun (58) determines internal stress from stress relaxation measurements and has also found internal stress to be equivalent to drying stress.

The effect of fiber orientation on the relationship between strength properties and drying stress is illustrated using the data of Fleischman (59). The variation of machine direction in-plane specific moduli with specific drying stress is shown in Fig. 14 for random, low, medium, and high levels of fiber orientation. At each orientation level the data points for wet pressing and wet straining fall on the same line.

The variation of machine direction tensile index with drying stress is shown in Fig. 15. It is interesting to note that the relationship is now non-linear with greater scatter at the medium and high levels of fiber orientation.

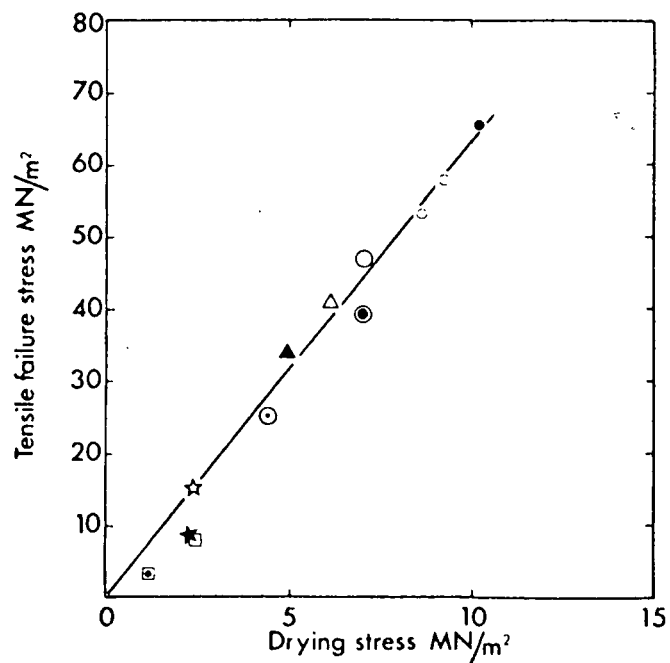


Figure 13. Variation of tensile stress with drying stress. Htun (58)

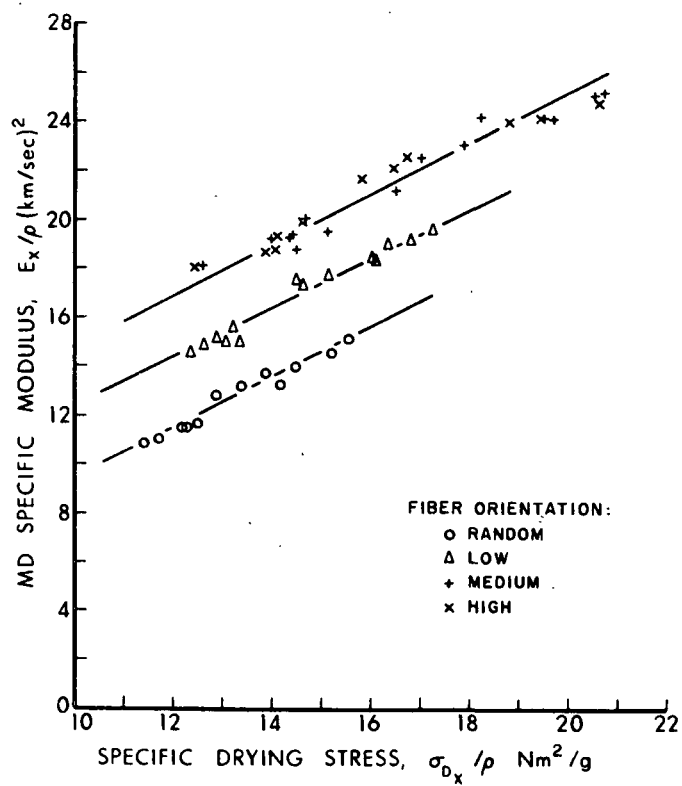


Figure 14. Modulus variation with drying stress based on data of Fleischman (59).

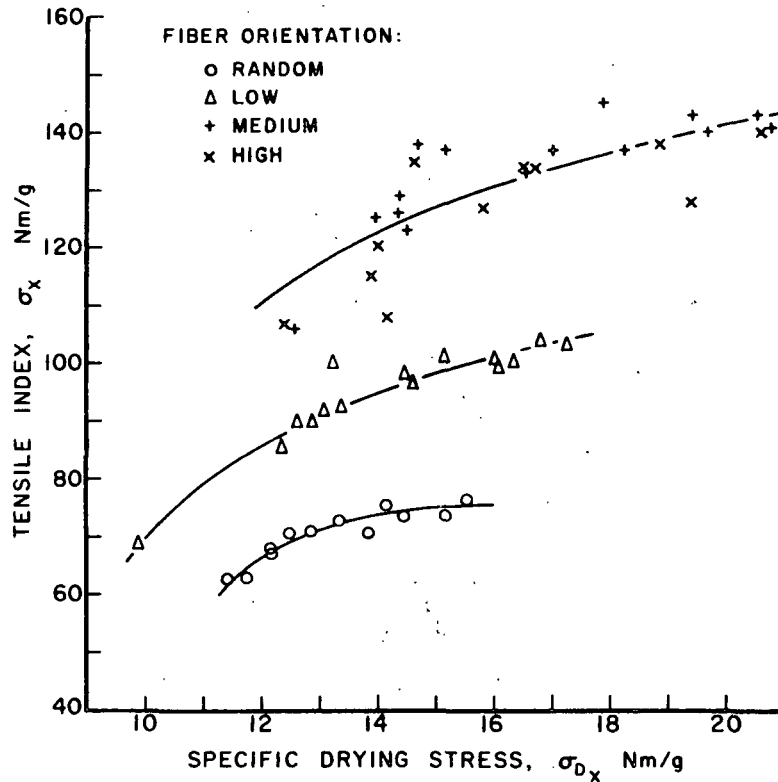


Figure 15. Tensile index variation with drying stress based on data of Fleischman (59).

However, if we plot the variation of tensile index with apparent density for different levels of wet straining and fiber orientation, as shown in Figure 16, then we see that there is approximately a linear relationship between tensile index and density at each of these levels.

5.0 Predictions of Strength Properties.

Product design and optimization would be greatly facilitated by our ability to predict strength properties. A number of attempts have been made to predict the tensile strength of paper and more recently its compressive strength. Other properties such as out of plane shear and tensile strength have yet to receive attention. Recently, attempts have been made to establish failure criteria for paper when it is subjected to combined stress situations (34,35).

Continuum Approach - Natural Flaws.

Nissan (60) and later Balodis (61) both examined the application of Griffith's fracture theory to paper. This equation has already been given and discussed in the introduction, i.e., Equation 1. Nissan assumed that an adequate test of the theory would be to determine if the failure stress σ is proportional to the

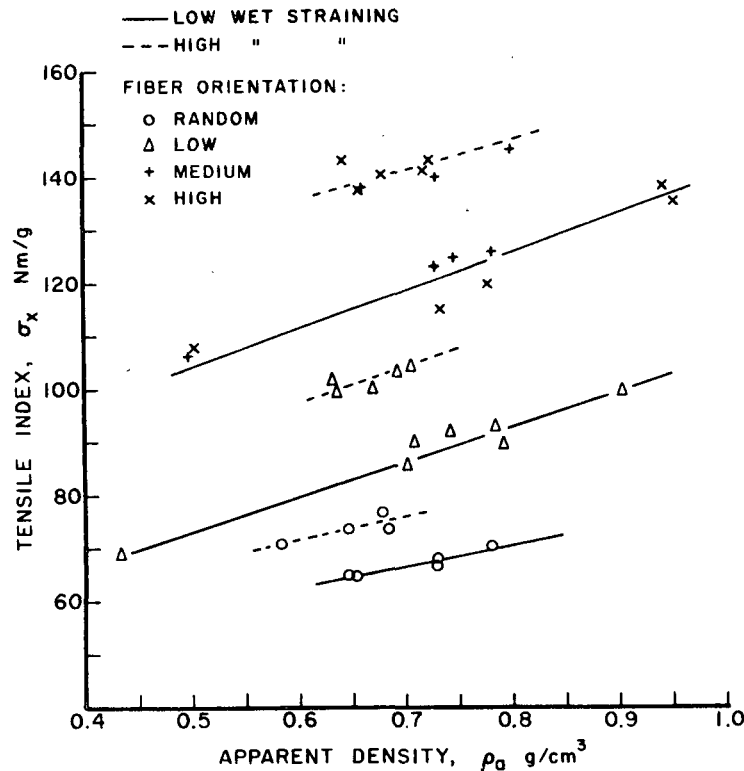


Figure 16. Tensile Index Variation with density based on data. Fleischman (59).

elastic modulus E raised to the half power, i.e. $\sigma \propto E^{\frac{1}{2}}$. This criteria assumes that the fracture energy and flaw size remain constant. Nissan found using Higgins' results for beaten pulps which had been acetylated that the power for the elastic modulus was 1.23. He therefore concluded that Griffith's theory did not hold and proceeded to develop his molecular approach based on hydrogen bonding.

Balodis (61) whose main concern was with measuring fracture energy also examined Griffith's theory. He argued that fracture energy would vary as modulus was varied and therefore looked at the relationship between failure stress σ and the product $(EG_c)^n$ for which he found $n=0.4$. He concluded, since his correlation coefficient was $r = 0.75$, this value of n was not significantly different from 0.5. No consideration was given by either of the authors to the conditions under which brittle fracture conditions might occur as discussed earlier in this paper or that the critical crack length might also vary for the conditions examined.

Seth and Page (62) have made careful measurements of fracture resistance and their results have been used to calculate a fracture resistance based on cellulose cross section, i.e. $(R \times 1.55 \times 9.81 \times 10^3)/BW$ having units of J/m² where R is their fracture resistance gcm/cm. The variation of fracture resistance with density, varied by refining, for the four pulps is shown in Fig. 17.

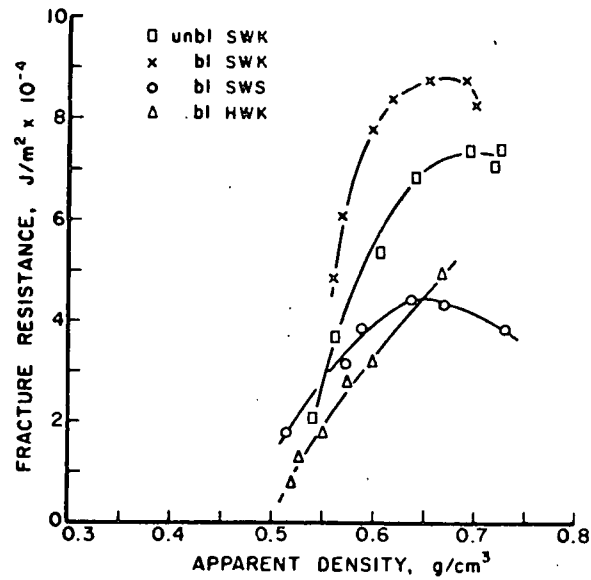


Figure 17. Variation of fracture resistance with density based on data of Seth and Page (62).

We note that there is a maximum in fracture resistance for three of the pulps. It would be interesting to determine how fracture resistance varies with wet pressing at constant levels of refining. The levels of fracture resistance are, according to Ashby (4), appropriate to ductile fracture as discussed earlier and it is unlikely that conditions for brittle fracture will be satisfied at Tappi standard conditions. The data of Seth and Page (62) have also been used to determine the index n , as discussed above, for the four pulps and the results are shown in Table VI.

TABLE VI

VALUES OF n FOR $\sigma^* = A (Gc/\rho E/\rho)^n$

Pulp Type.	A	n	r ²
Unbl. softwood kraft.	1.88	0.672	0.948
bl. softwood kraft.	0.517	0.847	0.944
bl. softwood sulfite.	0.969	0.798	0.894
bl. hardwood kraft.	4.62	0.476	0.970

In these calculations a specific fracture resistance Gc/ρ was used which is more consistent with the strength properties we have discussed so far. The correlations are considerably better than the one given by Balodis (61), and for the bleached hardwood kraft pulp the value of n is quite close to 0.5.

Network Predictions

Predictions for the ultimate tensile strength of paper developed by a number of researchers are summarized in Table VII. It is interesting that no fundamentally new contributions have been made in this area over the past fourteen years.

TABLE VII

PREDICTIONS FOR THE ULTIMATE STRENGTH OF PAPER. σ

Kallmes, O., Perez, M. (63)

1. Failure induced by bond failure

$$\sigma_B = \frac{1}{10} \frac{\sigma_f}{C} \frac{RBA}{RBA_{crit.}} \quad RBA \leq RBA_{crit.}$$

2. Failure induced by fiber failure

$$\sigma_F = \frac{1}{3} \sigma_f \left[1 - \frac{5}{6} C \frac{RBA_{crit.}}{RBA} \right] \quad RBA > RBA_{crit.}$$

$$\sigma_{max} = \sigma_{RBA_{crit.}} \rightarrow 0 = \frac{1}{3} \sigma_f$$

Perez, M. (64)

1. Release of microcompressions

$$\sigma = 0.35 \sigma_f RBA/RBA_{crit.}$$

Page, D. H. (65)

1. Fiber failure

$$\sigma = \frac{1}{6} \sigma_f \frac{RBA}{RBA_{crit.}} \left[\frac{1}{1 + \frac{1}{2} RBA/RBA_{crit.}} \right]$$

$$\sigma_{max} = \sigma_{RBA_{crit.}} \rightarrow 0 = \frac{1}{3} \sigma_f$$

where σ_f is fiber ultimate tensile stress, $RBA_{crit.} = t_f/\lambda W s$ (modified form of parameter introduced by Kallmes and Perez) t_f is fiber strength, λ fiber length, W fiber perimeter, s bond strength/unit area. C is a constant.

The equations developed by Kallmes and Perez (63) and Page (65) are rather similar as has been recently pointed out by Williams (66). Both predict that the tensile strength of the network approaches one third the tensile strength of its constituent fibers for a well bonded network. Page's equation has the

advantage that it does not contain any undetermined constants. It is also consistent with his and Seth's more recent work (67) that the elastic and plastic deformation behavior of the network is essentially controlled by fiber deformation behavior.

Page (65) made an impressive and largely successful attempt to validate his equation; however, some questions and criticism still remain. The form of the equation is such that a linear relationship should exist between inverse tensile strength and inverse relative bonded area (R.B.A.), provided that the bond to fiber strength ratio remains constant. Furthermore, it is assumed that this ratio will remain constant with wet pressing. As far as I am aware there have been no published results verifying, or otherwise, this basic relationship employing independent measurements of R.B.A. The measurement of R.B.A. is not particularly straightforward; for example, how does one measure the total surface area appropriate to a particular state of bonding? The most direct approach to this problem so far has been the work of Rennel (68,69) using nitrogen adsorption techniques to determine the total surface area of individually (sprayed) dried fibers. Rennel found that the total surface area varied with refining and that at low levels of refining the uncollapsed lumen contributed significantly to surface area. These difficulties have prompted some researchers (11,70) to replace R.B.A. with relationships involving apparent density.

El-Hosseiny (70) has recently made a critical examination of these strength theories and feels that, in view of their shortcomings, more refined theories are needed to understand the basics of paper strength. In his review he substituted apparent density for R.B.A. in the Page and Kallmes equation as well as deriving empirical equations based on either R.B.A. or apparent density, zero span, and fiber length to fiber wall thickness ratio, and found them to be equally good predictions of tensile strength. Although El-Hosseiny (71) demonstrates that there is a high degree of correlation between scattering coefficient and density, there is still the question of how to determine S_0 or the constants k_1 and k_2 of his Equation 3 (71) given below:

$$\text{R.B.A.} = k_1 + k_2 \rho_a, \quad (4)$$

where ρ_a is the apparent density.

The data of Rennel (72,76) has also been used by this author to demonstrate that there is indeed a reasonably linear relationship between R.B.A. and ρ_a as shown in Fig. 18 and Table VIII, although the r^2 values are not particularly impressive. The four pulp types A, B, C, and D are identified in Table X.

This author has also examined the dependence of ultimate strength on apparent density (73,74) and has found the following empirical correlation to hold quite well.

$$\begin{aligned} \sigma^* &= \sigma_o^* (\rho_a / \rho_f)^n \\ \text{or } \sigma^* &= \sigma_o^* (1 - \epsilon)^n \end{aligned} \quad (5)$$

TABLE VIII

RBA - DENSITY CORRELATION USING DATA OF RENNEL (72,76)

A. %RBA = $-31.2 + 125.6 \rho_a$.	$r^2 = 0.936$
B. %RBA = $8.208 + 77.71 \rho_a$.	$r^2 = 0.899$
C. %RBA = $-25.0 + 105.7 \rho_a$.	$r^2 = 0.918$
D. %RBA = $-21.1 + 81.7 \rho_a$.	$r^2 = 0.792$

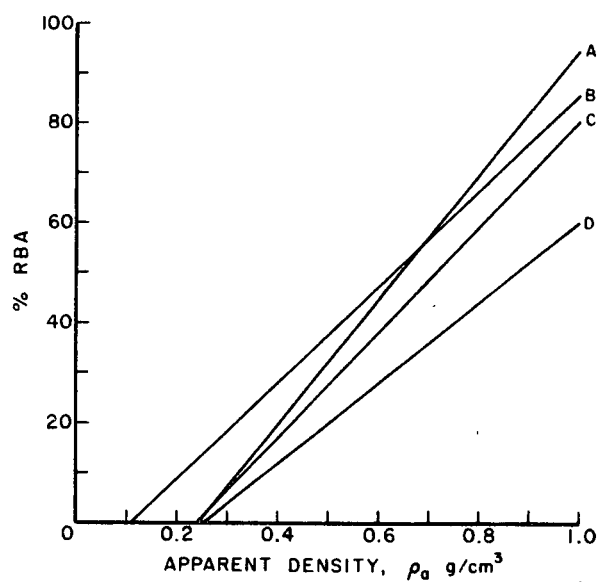


Figure 18. Variation of RBA with density based on data Rennel(72,76).

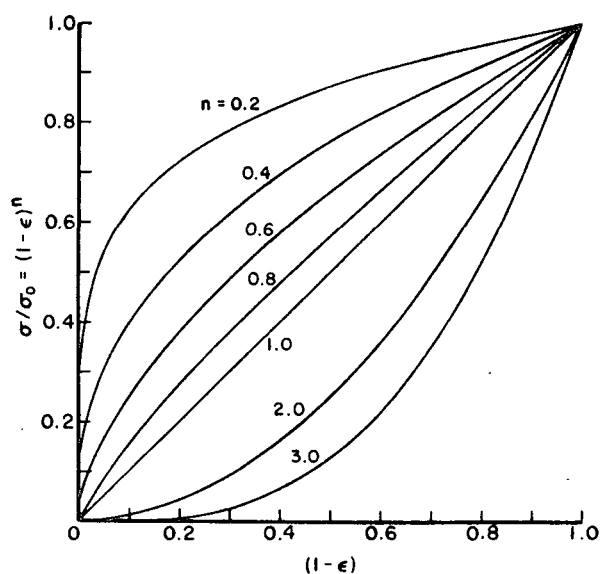


Figure 19. Tensile strength density relationship.

TABLE IX

PARAMETERS FOR $\sigma = \sigma_0(1-\epsilon)^n$ LUNER, et al.'s DATA (75)

Pulp I 19.0% Lignin				Pulp II 12.0% Lignin				Pulp III 2.3% Lignin			
C.S.F.	σ_0 Nm/g	n	r ²	C.S.F.	σ_0 Nm/g	n	r ²	C.S.F.	σ_0 Nm/g	n	r ²
680	125	1.39	0.995	690	143	1.36	0.985	660	124	1.05	0.992
680	105	1.08	0.998	680	137	1.06	0.986	640	118	0.809	0.992
590	138	1.04	0.992	580	145	0.921	0.969	590	131	0.839	0.982
410	126	0.890	0.994	410	151	0.796	0.962	370	136	0.637	0.949

r² = (correlation coefficient)²
Pulps I, II and III bleached birch.

TABLE X

PARAMETERS FOR $\sigma = \sigma_0(1-\epsilon)^n$ RENNEL'S DATA (72,76)

	Pulp A			Pulp B			Pulp C			Pulp D		
PFI Mill	σ_0 Nm/g	n	r ²	σ_0 Nm/g	n	r ²	σ_0 Nm/g	n	r ²	σ_0 Nm/g	n	r ²
unbeaten	152	1.20	0.982	133	0.991	0.985	16.2	0.449	0.981	106	0.879	0.857
500	164	1.10	0.997	143	0.947	0.952	-	-	-	83.3	0.879	0.827
1000	169	1.06	0.970	192	1.21	0.959	-	-	-	77.0	0.842	0.773
2000	169	0.973	0.967	169	1.05	0.965	53.0	0.651	0.981	73.9	0.784	0.953
4000	157	0.758	0.886	118	0.494	0.889	66.1	0.649	0.839	78.1	0.843	0.988
8000	150	0.598	0.777	126	0.513	0.971	84.0	0.758	0.929	75.6	0.828	0.979
16,000	122	0.235	0.492	122	0.472	0.977	81.2	0.539	0.898	73.4	0.847	0.959
32,000	165	0.735	0.936	125	0.466	0.943	81.6	0.792	0.879	60.3	0.615	0.943

r² = (correlation coeff.)²
Pulp A: Scandinavian Spruce
Pulp B: Douglas Fir
Pulp C: Dried Sulfate
Pulp D: Groundwood

The form of the equation is shown in Fig. 19. At a given network solidity ($1-\epsilon$), n decreases as tensile strength increases. Some results using the data of Luner (75) and Rennel (72,76) are summarized in Tables IX and X, respectively. The r^2 values obtained for Luner's data are quite good. The index n decreases with refining and decreasing lignin content, and might possibly be interpreted as an index of fiber flexibility. The ultimate strength of the network (i.e., when there are no voids) is fairly constant, having its highest value at a lignin content of 12.0%. Although some of the r^2 values for Rennel's data are rather poor (Table X), similar trends in the parameters σ^*_0 and n with refining were found.

Using Rennel's (76) R.B.A. data (gas adsorption), correlations between $1/(8/9)z$ and $1/RBA$ were also performed, (i.e., Page's equation) and the results are summarized in Table XI.

The r^2 values are similar to these obtained for the density correlations; however, reciprocals of the intercept values, i.e., $(8/9z)$ vary quite considerably, particularly for Pulp B. These results do not necessarily invalidate Page's equation, since the fiber to bond strength ratio may not be constant with wet pressing. The most direct approach, of course, would be to make independent measurements of the necessary fiber, bonding, and network parameters and then compare predicted strength values with measured ones.

Edgewise Compressive Strength σ_c

The measurement of compressive strength has received a good deal of attention recently particularly in view of its importance to container board performance. This activity has also been followed by attempts to understand the mechanisms of compressive failure and to predict compressive strength. The mechanism of failure is basically viewed as an instability phenomenon and as such is more amenable to analysis than tensile failure discussed in the last section.

The three general modes of failure shown in Fig. 20 which have been given by Uesaka and Perkins (77) are: 1) bending shear buckling, 2) shear band formation, and 3) bulging mode failure.

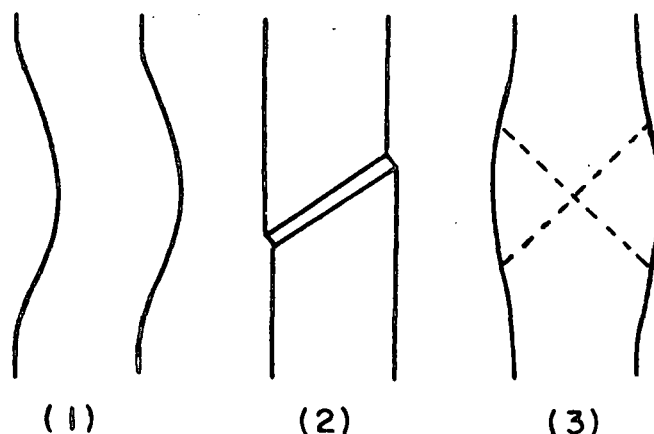


Figure 20. Compressive modes of failure, Uesaka and Perkins (77).

The precise conditions under which any one of these modes may occur have not yet been defined, although the above authors have established predictions for compressive strength for each of the modes. Their usefulness is limited by lack of experimental data for slide modulus. This approach treats paper and board as an orthotropic continuum. The observations of Fellers et. al (78) indicate fiber segment buckling is predominant in low density boards, while cell wall failure is in evidence at medium and high densities. Sachs and Kuster (79) observed the growth of voids and failure, and slippage between the S_1 and S_2 layers (during compressive loading) leading to interfiber bond failure and delamination. Fiber buckling and the establishment of the modes mentioned above were considered to be post failure phenomena. The growth of voids and delamination has also been pointed out by Habeger and Whitsitt (78) and is embodied in their predictions for compressive strength in contrast to Uesaka and Perkins who use a wholly continuum approach. The model adopted by Habeger and Whitsitt is that of an initially curved lamina bending into a reinforcing orthotropic medium. When the shear strength between this critical lamina (whose thickness is of the order of one fiber) and medium is exceeded, failure occurs, resulting in a large deformation. Buckling as such does not occur. The model gives results which agree in form with the empirical relationships which prompted its development as illustrated in Fig. 21. Furthermore, the plateau region (i.e., the span/caliper ratio over which the measured compressive strength is constant) is correctly predicted. Equations for the prediction of compressive strength are summarized in Table XII.

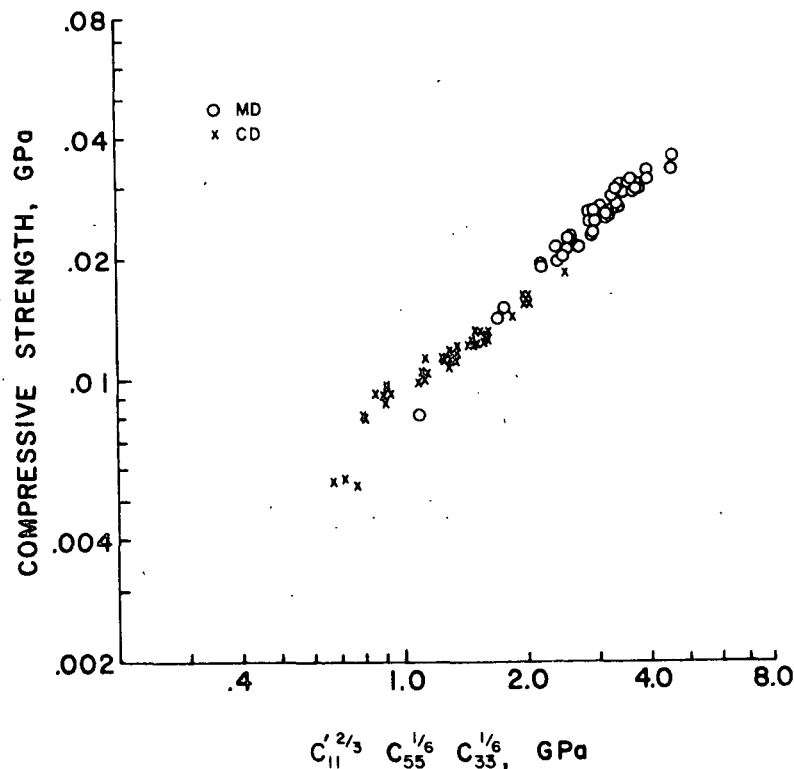


Figure 21. Compressive strength correlation. Habeger and Whitsitt (80).

TABLE XII

SUMMARY OF EQUATIONS FOR COMPRESSIVE STRENGTH.

Perkins, R. W. Jr., McEvoy, R. P. (82)

$$\sigma_c = 0.91L/[1 + 3/\pi^2 (1/t)^2 0.91 L/E_T]$$

Where L is Biot's slide modulus, E_T effective tangent modulus and $(1/t)$ span to thickness ratio.

Uesaka, T., Perkins, R. W. Jr. (77)

$$\sigma_c = k L$$

$$\sigma_c = 2/3 G/[1 + 3/\pi^2 (1/t)^2 2/3 G/E]$$

Where G is the out-of-plane shear modulus.

Habeger, C. C., Whitsitt, W. J. (80)

$$\sigma_c = \frac{\begin{pmatrix} 0.454 \\ 0.360 \end{pmatrix} C_{11}^{2/3} C_{55}^{1/6} C_{33}^{1/6}}{RW \left[1 - \frac{C_{13} + C_{55}}{C_{11}^{1/2} C_{33}^{1/2}} \right]}$$

where the roughness weakness factor $R.W. = (\delta A_0/t)(C_{55}/\tau_f)$.
and the C_{ij} 's are the orthotropic elastic constants. τ_f is the shear strength
and $(\delta A_0/t)$ the initial deflection to thickness ratio of the critical lamina.

A good approximation to the above equation is $\sigma_c \propto C_{11}^{2/3} C_{55}^{1/6} C_{33}^{1/6} / R.W.$

The constants in parenthesis of the above equation refer to the middle and outside locations of the critical lamina.

6.0 Ultimate Strength Requirements of Paper Products

Many of the measured properties of paper are inappropriate to its converting and end use. Uniaxial tensile strength might be expected to be an important property with respect to runnability (web breaks). However, when webs

contain flaws such as shives (83) and edge cracks, failure is likely to occur at stresses considerably below its maximum tensile stress. One promising approach to this problem area has been that of Page and Seth (84) who have demonstrated the validity of the fracture mechanics approach. A technique was developed for making meaningful fracture resistance measurements and a correlation, albeit a very time consuming one, was established between fracture resistance and runnability.

Fracture in an unflawed sheet will be initiated in a region of high stress concentration and will rapidly propagate if sufficient energy is stored in the web. Stress concentration may arise due to non-uniformities in sheet structure (partial bond failure may be viewed as one means by which stress concentration in the network is alleviated) and how the web is stressed. Although non-uniformities in structure, i.e., mass and strength, have been the concern of a number of researchers (22,85,86), the impact of mass and strength non-uniformities on runnability have yet to be determined.

The safe transportation and storage of goods and materials in corrugated containers requires amongst other things that they are able to withstand top to bottom compressive loading. Compressive strength, so it appears, is on considerably firmer ground in the area of box design. Some years ago McKee, et al. (87) established a relationship between box compressive strength, P , and the compressive strength and flexural stiffness of combined board, and box geometry.

McKee et al., (87)

$$P = 2.028 P_m^{0.746} (D_x D_y)^{0.254} Z^{0.492}$$

Where P_m is edgewise compressive strength, D_x and D_y , are the flexural stiffnesses in MD and CD direction of combined board, and Z is the box perimeter.

A precise relationship between the compressive strength of combined board and its components, the liner and medium, has yet to be established. The compressive strength of the components is clearly important and in the past ring crush has been used as a relatively simple measure of it; however, as reviewed earlier, a number of methods are available for the measurement of intrinsic compressive strength. The simple summation model (88) and the more fundamental work of Johnson and Urbanek (89,90) are possible approaches to developing working relationships between combined board and component compressive strength.

The importance of interlaminar shear deformation in practice has yet to be fully appreciated. In cases where other stresses are present its precise role may be difficult to determine. In forming corrugating medium out-of-plane shear deformation occurs, and bending, tensile, and compressive stresses are

also present. It is possible to delaminate the medium under certain running conditions, and shear deformation (possibly low shear strength) must be a contributing factor. Shear deformation occurring during flute formation might also adversely affect compressive strength; however, this does not seem to be the case (39). Nevertheless, shear in combination with other stresses might have a more adverse effect on the compressive strength performance of the medium.

Calendering and supercalendering are other areas where out-of-plane shear deformation could be important. It has been proposed that paper undergoes significant shear deformation in the supercalender nip, and usually there is a loss of tear particularly C.D., while tensile strength may either increase or decrease (91). No good explanation for these effects has yet been presented. Compressive strength, adhesive joints, and surface strength are areas where out-of-plane properties are important. Surface strength is important in printing processes where localized normal or Z direction stresses are present. Sometimes failure will occur due to local imperfections such as vessel elements. Adhesive joints, e.g., medium and liner joints in corrugating etc., are complicated by the application of adhesive which can modify the surface layers of the adherends and if aqueous based can cause interfacial weaknesses (92). Nevertheless the so-called cohesive failure of the adherends when subjected to out-of-plane combined stresses is an important area of consideration. The use of polymer reinforced paperboard materials for shoe innersoles is an interesting example from the speciality paper field. In Welt rib construction the innersole, which forms the 'keel' of the shoe, has a rib cemented to its underside. The upper material of the shoe is attached to this rib by side lasting and stitching operations. During these operations depending on shoe style etc., a complex dynamic loading is imparted to the rib structure through the upper material which can lead to adhesive failure, or in some cases cohesive failure of the innersole board material. This type of failure in shoe production can be quite costly.

Paper is a unique material. Over the years attempts have been made with varying degrees of success to apply other disciplines of materials science to paper, i.e., continuum mechanics, composites, fracture mechanics, etc., to help in providing a more basic and rigorous understanding of its behavior. In principle we would like to be able to design a paper product from first principles, i.e., determine the stresses which it will be subjected to during converting and end use such that the most suitable combination of materials may be selected. The nature of paper and papermaking are such that this ambition may never be totally realized.

I realize that I have omitted a number of important areas in this presentation. However I hope that my brief overview of the ultimate strength properties of paper represents a reasonably accurate statement of the current state of our paper physics art and that the stage has been set for the papers which follow in this session.

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